

An Enhanced Particle Swarm Optimization with Mutation for Mean-Value-at-Risk Portfolio Optimization in the Indonesian Banking Sector

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Abstract

Portfolio optimization in emerging markets is challenging because high volatility and non-normal return distributions reduce the effectiveness of traditional mean–variance models, which tend to underestimate downside risk. This study aims to develop and evaluate an Enhanced Particle Swarm Optimization with Mutation (PSO with Mutation) for portfolio optimization under the Mean-Value-at-Risk (Mean-VaR) framework in the Indonesian banking sector. The novelty of this approach lies in integrating a mutation operator into standard PSO to maintain population diversity, prevent premature convergence, and improve exploration of the solution space. To evaluate the method, daily adjusted closing prices of 31 Indonesian bank stocks from January 2020 to July 2025 were collected. Preprocessing included removing tickers with incomplete data and computing daily returns. The optimization problem was formulated using Mean-VaR as the risk measure, with portfolio weight constraints. The proposed PSO with Mutation was benchmarked against standard PSO, Genetic Algorithm (GA), Bat Algorithm (BA), BA with Mutation, and classical models (Markowitz and Monte Carlo–based VaR). Performance was assessed using expected return, Mean-VaR, risk-adjusted return, Sharpe ratio, execution time, and stability across 25 independent runs. The results show that PSO with Mutation achieved a competitive expected return (0.0020), the lowest Mean-VaR (0.0311), the highest risk-adjusted return (0.0650), and the lowest variability across runs, while maintaining acceptable execution time. These findings confirm that mutation-enhanced PSO provides a robust, balanced, and efficient solution for portfolio optimization, making it highly relevant for investors in volatile emerging markets and advancing research on hybrid metaheuristics in financial optimization.

Keywords : Indonesian Banking Sector, Mean Value-at-Risk, Mutation Operator, Particle Swarm Optimization, Portfolio Optimization.

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1. INTRODUCTION

Portfolio optimization offers significant advantages for investors by maximizing returns while managing risk. Using models like the Mean-Variance framework (Markowitz), investors can construct portfolios that either increase returns for the same risk or reduce risk for the same return, with added benefits such as tax efficiency [1], [2]. Portfolio optimization is critical in emerging markets when volatility and uncertainty challenge traditional investment strategies. While Markowitz's mean-variance theory [3] provides a foundation for balancing risk and return, it often underperforms in high-risk environments. Recent advances, such as Genetic Algorithms (GA), Machine Learning (ML) [4], [5], advanced hedging [6] and financial indicators [7], have improved portfolio adaptability. These trends highlight the importance of hybrid methods that combine classical models with AI-based approaches to optimize portfolios in dynamic, information-inefficient markets.

Traditional Mean-Variance Optimization (MVO), introduced by Markowitz, balances expected returns against variance but assumes normally distributed returns and treats upside and downside risks

equally. This symmetric view fails to account for investors' greater aversion to losses, often leading to suboptimal outcomes during market downturns, especially in volatile emerging [8], [9], [10], [11]. To address this, downside risk measures such as Value-at-Risk (VaR) and Conditional VaR (CVaR) have been adopted. These offer more accurate loss estimations under extreme conditions by focusing specifically on potential losses beyond normal expectations [12].

The Mean-VaR model enhances portfolio optimization by combining expected returns with VaR, offering a more investor-aligned, loss-sensitive framework. It explicitly accounts for downside risk, aligning with behavioral finance findings that investors prioritize loss avoidance [13], [14]. Research shows that portfolios optimized with Mean-VaR outperform traditional MVO during periods of financial stress, providing greater stability and capital preservation [15], [16]. These strengths make Mean-VaR a robust and practical choice for portfolio optimization in uncertain and risk-sensitive environments.

The Mean-VaR portfolio optimization problem is inherently a global optimization problem due to its non-linear, non-convex, and multi-dimensional nature. Traditional optimization methods often struggle in this landscape, frequently converging to local optima, particularly when asset returns are skewed or exhibit heavy tails [17], [18]. Particle Swarm Optimization (PSO), a metaheuristic algorithm inspired by swarm intelligence, offers a compelling solution. It is renowned for its simplicity, requiring minimal parameter tuning, and fast convergence, enabling timely responses to dynamic market changes [19], [20]. PSO simulates social behaviour among particles, allowing both global exploration and local refinement, which is crucial for navigating complex risk-return surfaces [21], [22]. PSO also supports multi-objective optimization, making it ideal for balancing expected returns against downside risk in the Mean-VaR framework [23]. These features make PSO a robust and efficient tool for solving the Mean-VaR problem and optimizing portfolios under uncertainty [24], [25], [26].

PSO is very effective in solving diverse optimization problems including portfolio selection. However, a major limitation of standard PSO is premature convergence, where particles settle into suboptimal regions early in the search, limiting exploration in complex or multimodal landscapes [27]. To overcome this, mutation is introduced to enhance swarm diversity. By adding small, normally distributed perturbations to particle positions, the algorithm escapes local optima and explores new regions of the search space [28]. This stochastic diversification improves PSO's ability to find global optima and maintain a healthy balance between exploration and exploitation. Moreover, mutation increases robustness in dynamic environments, such as fluctuating financial markets. It allows PSO to adapt rapidly to changing optima, improving responsiveness to real-time data [29]. Research confirms that PSO with mutation achieves faster convergence, better solution quality, and higher resilience against local traps compared to classical PSO [27], [28]. Thus, incorporating mutation into PSO significantly enhances its performance and suitability for complex portfolio optimization tasks.

This framework is applied within the context of emerging markets, where investment environments are often characterized by both high return potential and significant risks stemming from volatility, regulatory changes, and geopolitical factors [30], [31]. In particular, the Indonesian banking sector serves as a strategic case study due to its central role in national economic development, resilience to global economic shocks, and increasing adoption of digital transformation [32]. The sector is further distinguished by high market capitalization, strong liquidity, and a critical economic function, mobilizing capital, supporting investment, and ensuring macroeconomic stability [32], [33]. Additionally, Indonesian banks foster financial inclusion and entrepreneurial growth, contributing to poverty reduction and long-term social development [33], [34].

Previous studies have explored portfolio optimization using approaches such as the Genetic Algorithm (GA), Bat Algorithm (BA), and standard Particle Swarm Optimization (PSO) under Mean-Variance or Mean-VaR frameworks [35], [36], [37], [38]. While these methods demonstrate the usefulness of metaheuristics in financial optimization, they are often limited by premature convergence,

sensitivity to heavy-tailed return distributions, and inconsistent performance across multiple runs. Moreover, most prior research has focused on developed markets or relied on conventional models without explicitly addressing the unique characteristics of emerging markets such as Indonesia, where volatility and downside risks are more pronounced.

To address this gap, the present study proposes an Enhanced PSO with Mutation for Mean-VaR portfolio optimization in the Indonesian banking sector. Mean-VaR is adopted as a more robust risk measure than variance, particularly relevant for capturing downside risk in volatile emerging markets. Unlike earlier studies that primarily employed GA, BA, or standard PSO, this work introduces a mutation operator into PSO to preserve population diversity, reduce the risk of premature convergence, and enhance exploration. The proposed algorithm is applied to Mean-VaR-based portfolio optimization, addressing the shortcomings of traditional Mean-Variance models and demonstrating superior performance through a comparative analysis against PSO, GA, BA, and BA with Mutation. To the best of our knowledge, this is the first study to apply a mutation-enhanced PSO to Mean-VaR portfolio optimization in the context of the Indonesian banking sector, providing both improved robustness and practical applicability.

2. METHOD

This study proposes an enhanced PSO algorithm integrated with a mutation operator to address portfolio optimization under Mean-VaR constraints. The proposed approach is tailored to effectively manage the nonlinear, volatile, and asymmetric nature of financial return distributions, particularly within the dynamic environment of the Indonesian banking sector. By incorporating a mutation operator, the algorithm enhances global exploration and reduces the risk of premature convergence, thereby improving the search for optimal asset allocations. The overall research methodology is illustrated in Figure 1.

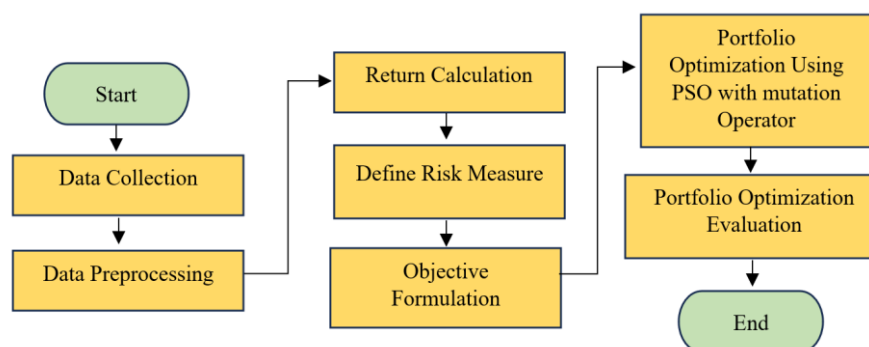


Figure 1. Flowchart of the research method

2.1. Data Collection

This study utilizes historical daily stock price data from 36 Indonesian banking institutions, such as Bank Central Asia (BBCA), Bank Rakyat Indonesia (BBRI), Bank Mandiri (BMRI), Bank Negara Indonesia (BBNI), etc. The data were retrieved from Yahoo Finance, covering the period from January 2020 to July 2025. Each stock price entry comprises several features, namely the Date (recording day of the data), Open (price at market opening), High (highest trading price of the day), Low (lowest trading price of the day), Close (final trading price at market close), Adjusted Close (closing price adjusted for corporate actions such as stock splits and dividends), and Volume (total number of shares traded, reflecting market activity). In this study, only Adjusted Close prices are employed for portfolio optimization because they incorporate the effects of corporate actions (e.g., stock splits, dividends, and rights issues), thereby providing a consistent and accurate basis for historical return calculations. This

adjustment is essential to avoid distortions that may arise from raw price data and to ensure reliable estimates of expected returns, variances, and covariances which are parameters fundamental to Modern Portfolio Theory (MPT) and Mean–Variance Optimization (MVO).

2.2. Data Preprocessing

The preprocessing stage in this study was conducted to ensure the quality and consistency of the historical return data used for portfolio optimization. Specifically, the preprocessing involved removing bank stocks with incomplete data over the observation period. These banks were excluded to maintain analytical integrity, as optimization algorithms require complete and comparable historical data across all assets. No interpolation, imputation, or other transformation techniques were applied, ensuring that the results accurately reflect actual market conditions. Following data preprocessing, a total of 31 bank stocks with complete historical return data were selected for the portfolio optimization analysis. The list of these banks and their corresponding stock tickers is presented in Table 1.

Table 1. List of banks and their stock tickers

No.	Ticker	Bank Name
1	AGRO.JK	Bank Raya Indonesia (BRI Agro)
2	ARTO.JK	Bank Jago
3	BABP.JK	Bank MNC Internasional
4	BBCA.JK	Bank Central Asia (BCA)
5	BBHI.JK	Allo Bank Indonesia
6	BBNI.JK	Bank Negara Indonesia (BNI)
7	BBRI.JK	Bank Rakyat Indonesia (BRI)
8	BBYB.JK	Bank Neo Commerce
9	BCIC.JK	Bank Capital Indonesia
10	BDMN.JK	Bank Danamon Indonesia
11	BEKS.JK	Bank Bumi Arta (eks Pundi)
12	BGTG.JK	Bank Ganesha
13	BJBR.JK	Bank Jabar Banten (BJB)
14	BJTM.JK	Bank Jatim
15	BKSW.JK	Bank QNB Indonesia
16	BMRI.JK	Bank Mandiri
17	BNBA.JK	Bank Bumi Arta
18	BNGA.JK	Bank CIMB Niaga
19	BNII.JK	Bank Maybank Indonesia
20	BPII.JK	Bank Panin Dubai Syariah
21	BRIS.JK	Bank Syariah Indonesia (BSI)
22	BSWD.JK	Bank Sahabat Sampoerna
23	BTPS.JK	Bank BTPN Syariah
24	BTPN.JK	Bank BTPN
25	BBTN.JK	Bank Tabungan Negara (BTN)
26	MEGA.JK	Bank Mega
27	MCOR.JK	Bank China Construction Indonesia
28	MAYA.JK	Bank Mayapada
29	NISP.JK	Bank OCBC NISP
30	NOBU.JK	Bank National Nobu
31	SDRA.JK	Bank Woori Saudara Indonesia

2.3. Return Calculation

In this study, daily simple returns were calculated using the adjusted close prices of each bank stock. The adjusted close price reflects all relevant corporate actions such as dividends and stock splits, thus providing an accurate basis for return estimation. The simple return for each stock on day t is calculated by Equation (1),

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 \quad (1)$$

where r_t represents daily return on day t . P_t and P_{t-1} represent Adjusted Close price at day t and Adjusted Close price on the previous trading day. This calculation was applied to each of the 31 retained bank stocks, resulting in a daily return matrix of size $T \times N$, where T is the number of trading days and N is the number of stocks.

These simple return values serve as the foundation for subsequent computations in the portfolio optimization framework. The several financial metrics, such as expected return, Mean-VaR and Sharpe Ratio, are derived from the daily return series.

2.4. Define Risk Measure

Mean-VaR is employed in this study as the primary measure of downside risk due to its effectiveness in capturing extreme loss behavior under uncertain market conditions. This metric is particularly suited to financial portfolios in emerging markets, where return distributions frequently exhibit skewness, fat tails, and elevated downside exposure. By concentrating on the lower tail of the return distribution, Mean-VaR aligns more closely with investor preferences for capital preservation during periods of market turbulence. Unlike traditional variance-based measures that treat gains and losses symmetrically, Mean-VaR focuses exclusively on adverse outcomes. Specifically, it represents the average of returns falling below a specified percentile threshold (e.g., the 5th percentile at a 95% confidence level), offering a targeted assessment of downside risk.

Mean-VaR is computed as the average of all portfolio returns that fall below a specified quantile threshold, typically defined at a confidence level of 95%. Formally, the Mean-VaR at the 95% confidence level is computed as the average of all return observations that are less than or equal to the 5th percentile of the return distribution, i.e., the Value-at-Risk (VaR) at 95%, as defined in Equation (2),

$$\text{Mean-VaR}_\alpha = \mathbb{E}[R \mid R \leq \text{VaR}_\alpha] \quad (2)$$

where R denotes the portfolio return, VaR_α is the Value-at-Risk at confidence level α , and $\mathbb{E}[R \mid R \leq \text{VaR}_\alpha]$ represents the expected return conditional on exceeding the risk threshold. VaR_α is a confidence level α (e.g., 95%) estimates the maximum expected loss over a specific time horizon (e.g., daily, weekly) under normal market conditions, with a given probability. Given a time-series of historical portfolio or asset returns, begin by sorting the return values in ascending order. Then, select a confidence level α , typically set at 95%. The Value-at-Risk (VaR) at the α confidence level is calculated by identifying the $(1 - \alpha)$ -th percentile of the return distributio as expressed in Equation (3).

$$\text{VaR}_\alpha = \text{Percentile}_{(1-\alpha)}(R) \quad (3)$$

$\text{VaR}_{95\%}$ is the 5th percentile of return distribution. This means that there is a 5% chance that the portfolio will lose more than the VaR value on a given day.

2.5. Objective Formulation

In this study, the portfolio optimization problem is formulated as a constrained optimization task under the Mean-VaR framework, which seeks the best allocation of asset weights that balances profitability with downside risk. Let $\mathbf{w} = (w_1, w_2, \dots, w_n)$ denote the portfolio weights with n assets, and $R_{p,t} = \mathbf{w}^T \mathbf{r}_t$ is the realized portfolio return at time t , where $\mathbf{r}_t = (r_{1t}, r_{2t}, \dots, r_{nt})^T$ is the vector of asset returns at time t . The expected portfolio return is then defined by Equation (4),

$$\mu_p = \frac{1}{T} \sum_{t=1}^T R_{p,t} \quad (4)$$

where T is the number of observations.

Downside risk is measured using VaR at confidence level α , computed from the empirical distribution of $\{R_{p,t}\}_{t=1}^T$. The optimization can be expressed in two complementary forms which are weighted trade-off formulation and ratio-based formulation.

1) Weighted trade-off formulation

Ratio-based formulation is defined by Equation (5),

$$\text{Mean-VaR}_\alpha = \max_{\mathbf{w}} F(\mathbf{w}) = \mu_p - \lambda \cdot \text{VaR}_\alpha(R_p) \quad (5)$$

where λ is a risk-aversion parameter that controls the trade-off between maximizing return and limiting downside risk. $\text{VaR}_\alpha(R_p)$ is the VaR of the portfolio return R_p at confidence level α . It represents the maximum potential loss not exceeded with probability $1 - \alpha$. μ_p could also be represented by expected return portfolio $E[R_p]$.

2) Ratio-based formulation (fitness function for metaheuristics)

Ratio-based formulation is used for the fitness function which is defined in Equation (6). It is also called by minus of **risk-adjusted return**. This objective should be to be minimized.

$$\text{Fitness} = - \frac{\mu_p}{\text{Mean-VaR}_\alpha} \quad (6)$$

In this case, minimizing the fitness function is equivalent to maximizing return per unit of downside risk, ensuring that portfolios with higher returns and lower risks achieve better fitness values.

Both formulations are subject to the standard constraints in Equation (7),

$$\sum_{i=1}^n w_i = 1, \quad w_i \geq 0, \quad \forall i, \quad w_i \leq w_{\max} \quad (7)$$

ensuring full investment, non-negativity, and diversification control.

Mean-VaR quantifies average loss in the worst-case tail (bottom 5% of returns), defined as Equation (8),

$$\text{Mean-VaR}_\alpha = -E[R_p \mid R_p \leq \text{VaR}_\alpha] \quad (8)$$

where VaR_α is the return threshold at confidence level α .

2.6. Portfolio Optimization Using PSO with Mutation Operator

This study proposes an Enhanced PSO algorithm integrated with a mutation operator to solve the portfolio optimization problem using Mean-VaR as the downside risk measure. The objective is to find an optimal portfolio allocation that maximizes expected return while minimizing downside risk as defined as Equation (6). The constraints are $\sum w_i = 1$ (budget constraint) and $w_i \geq 0$ (no short selling).

Each particle in PSO represents a candidate solution (portfolio weights). The proposed enhancements are as follows:

a. Initialization

- Generate a swarm of particles with random feasible portfolio weights. Each particle in the PSO algorithm represents a candidate solution of portfolio weights $x_i^0 = w_i = (w_{i1}, w_{i2}, \dots, w_{in})$, $i = 1, \dots, Np$. Np represents the number of particles.
- Initialize velocity, personal best p_i^0 , and global best g^0 for each particle.
- Initialize $t = 0$

b. Velocity and Position Update

- The velocity and position updates at iteration $t + 1$ are defined as in Equations (9) and (10),

$$v_i^{t+1} = \omega v_i^t + c_1 r_1 (p_i^t - x_i^t) + c_2 r_2 (g^t - x_i^t) \quad (9)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (10)$$

where x_i^t is current position of particle i at iteration t , v_i^t is current velocity, p_i^t is best-known position of particle i (personal best), g^t is global best position among all particle, ω is inertia weight. c_1, c_2 are cognitive and social coefficients, respectively. $r_1, r_2 \sim U(0,1)$ are random values drawn from a uniform distribution.

- Apply normalization to keep weights feasible. Constraint handling is enforced by normalizing the weight vector to ensure $\sum w_i = 1$ and clamping values to stay within $[0,1]$.

c. Mutation Operator (Enhancement)

- With a small probability, mutation operator is applied to selected particles by Equation (11).

$$x'_i = x_i + \mathcal{N}(0, \sigma^2) \quad (11)$$

- It helps diversify the population, avoid premature convergence, and enhance global exploration.

d. Fitness Evaluation

- Calculate portfolio return, Mean-VaR, and fitness for each particle.
- Update personal and global bests accordingly.

e. Early Stopping

- The algorithm terminates when a predefined maximum number of iterations is reached or when convergence criteria are met.
- If no improvement in global best fitness for a set number of iterations (e.g., 100), stop early.

For each run, the algorithm outputs are optimal portfolio weights, expected return, Mean-VaR, Sharpe Ratio and Execution time. The algorithm is executed for **25 independent runs** to assess performance consistency. Comparative analysis is conducted with other algorithms (GA, standard PSO, BA, BA with Mutation).

f. Output

The final output of the PSO process is the optimal portfolio weight vector w^* that is global best particle (g^*) that minimize the risk-adjusted return. The optimized portfolio is then evaluated in terms of expected portfolio return, Mean-VaR, Risk-Adjusted Return and Sharpe ratio to assess its performance under realistic market scenarios.

2.7. Comparative Analysis and Evaluation

This section focuses on evaluating the effectiveness of the proposed Enhanced PSO with Mutation for portfolio optimization under Mean-VaR constraints (PSO with Mutation + Mean VaR) by comparing its performance with other commonly benchmark models in portfolio optimization and several Mean-VaR optimization approaches using metaheuristics. The goal is to assess whether PSO with Mutation offers superior results in terms of risk-adjusted performance, convergence behaviour, and computational efficiency.

1. Benchmark Methods for Comparison

The PSO with Mutation + Mean VaR is evaluated against several widely used benchmark models in portfolio optimization, namely:

- GA for Mean-Variance [35],
- Markowitz Model [36] and
- VaR + Monte Carlo [37].

The PSO with Mutation + Mean VaR is also benchmarked against several Mean-VaR optimization approaches using metaheuristics, including:

- Standard PSO (PSO) + Mean VaR (PSO + Mean VaR) [38],
- Genetic Algorithm (GA) + Mean VaR (GA + Mean VaR),
- Bat Algorithm (BA) + Mean VaR (BA + Mean VaR),
- BA with Mutation + Mean VaR (BA with Mutation + Mean VaR),

PSO, BA, and BA with Mutation are chosen due to their wide application in financial optimization tasks and their capabilities to solve global, nonlinear, and constrained optimization problems.

2. Evaluation Metrics

To ensure a comprehensive comparison, multiple evaluation criteria are used:

a. Expected Return

Expected return is calculated as the arithmetic mean of daily simple returns, representing the average performance of each asset over the observation period. It measures the average daily return of the optimized portfolio. The expected portfolio return μ_p is computed as Equation (4).

b. Mean-VaR

Mean usually refers to the expected return of the portfolio and VaR is a downside risk measure that represents the maximum potential loss over a given time horizon at a certain confidence level. Mean-VaR aims to maximize the expected return (Mean) and to minimize downside risk (VaR). Mean-VaR is defined by Equation (5).

c. Risk-Adjusted Return

A risk-adjusted return metric reflecting the ratio between excess return and volatility.

d. Sharpe Ratio

Sharpe ratio is a risk-adjusted performance metric calculated as the ratio between the asset's excess return (over a risk-free rate) and the standard deviation of returns, indicating how well the return compensates for the risk taken. It is used to measure the risk-adjusted return of an investment or portfolio. Sharpe ratio is defined by Equation (12).

$$SR = (R_p - R_f) / \sigma_p \quad (12)$$

where σ_p is portfolio risk that is defined by Equation (13).

$$\sigma_p = \text{sqrt}(g^T \times \Sigma \times g) \quad (13)$$

Σ represents covariance matrix of asset returns

e. Execution Time

It measures algorithmic efficiency.

3. Experimental Setup

Each algorithm is independently run 25 times on the same return dataset derived from historical stock prices of major Indonesian banks. The dataset is pre-processed to remove incomplete data. Each run uses the same portfolio constraints and parameter settings to ensure fair evaluation.

4. Statistical Analysis

The mean and standard deviation of each metric across the 25 runs are calculated and tabulated. This helps identify the consistency of algorithm performance, the robustness under stochastic optimization and the Trade-offs between risk and return across methods

3. RESULTS

This section presents the results of the study. The results are organized into three parts to ensure clarity and logical progression. First, we provide sample outputs from the data collection process to illustrate the raw structure of the Indonesian banking stock dataset. Second, we present sample outputs from the preprocessing stage, including daily return calculations, handling of missing values, and distributional analysis. These steps establish the foundation for the optimization stage by ensuring that the input data are consistent, comparable, and statistically characterized. Finally, we report the results of evaluation and comparative analysis of the proposed Enhanced PSO with Mutation for Mean-VaR portfolio optimization against established benchmarks. This structure allows us to move from raw data, through preprocessing, to performance evaluation in a systematic way.

3.1. Sample Outputs of Data Collection

Table 2 presents a sample of BBNI's daily stock data. Each row corresponds to one trading day and includes the opening price (Open), highest price (High), lowest price (Low), closing price (Close), adjusted closing price (Adj Close), and trading volume (Volume). In the five days shown (2–8 January 2020), the closing price declined from 3,887.5 to 3,712.5, the adjusted close was consistently lower, and trading activity ranged between 18.6 million and 40.9 million shares. Figure 2 shows the adjusted close price of BBNI.JK from January 2020 to July 2025. The trajectory captures movements across the observation period, including phases of decline, recovery, growth, and fluctuations.

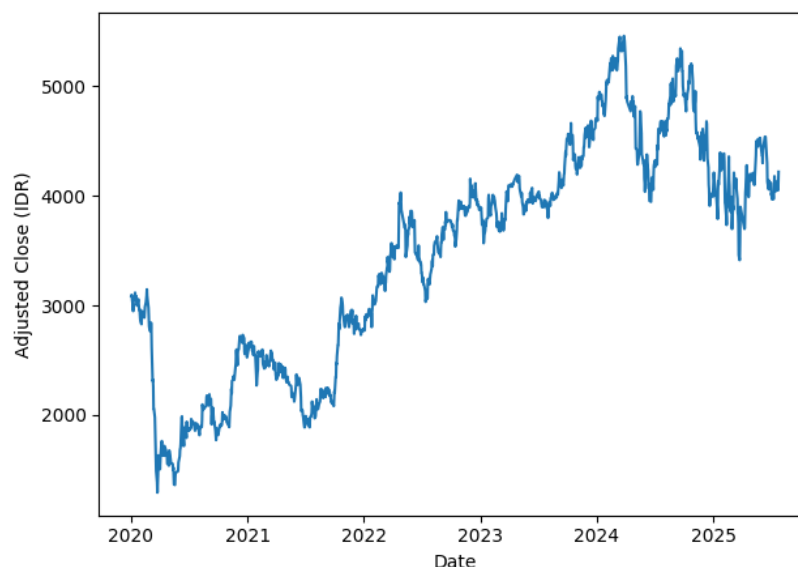


Figure 2. Example adjusted close trajectory (BBNI ticker)

Figure 4 shows a histogram of daily returns for BBNI.JK from January 2020 to July 2025. The distribution is more peaked and has heavier tails compared to the normal curve, with a thicker left tail (down to about -12%) and an extended right tail (up to approximately $+11\%$).

3.3. Results of Evaluation

This section shows the comparison of the results of the proposed Enhanced PSO with Mutation for Mean-VaR Portfolio Optimization against several established approaches. These include PSO + Mean-VaR [38], GA for Mean-Variance [35], the classical Markowitz model [36], VaR with Monte Carlo [37], and VaR with Bootstrapping. The classical Markowitz model has two scenarios which are LongOnly – Max Sharpe and LongOnly Min VaR. We also benchmark our method against standard metaheuristic variants, namely GA for Mean-VaR, BA for Mean-VaR, and BA with Mutation for Mean-VaR. In addition, results from prior studies are referenced to provide context and validation. Each algorithm was run independently 25 times, and performance was assessed based on average expected return, Mean-VaR, risk-adjusted return, Sharpe ratio, and execution time. Figures 5–9 present a visual summary of these performance metrics.

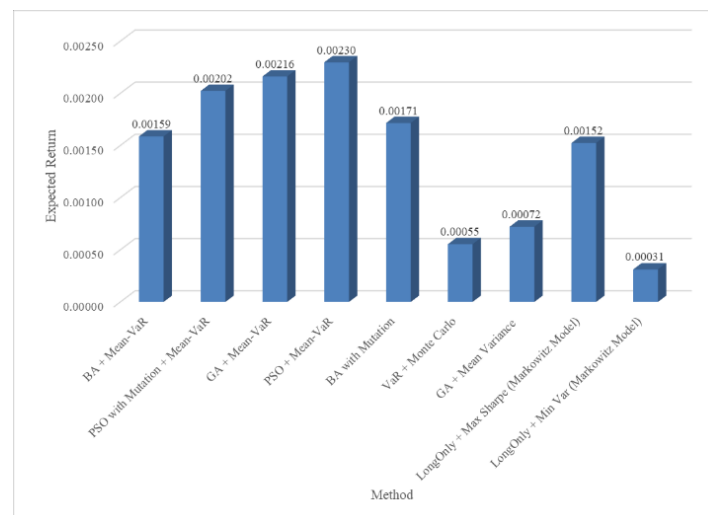


Figure 5. The average of expected return by resulted by portfolio optimization methods

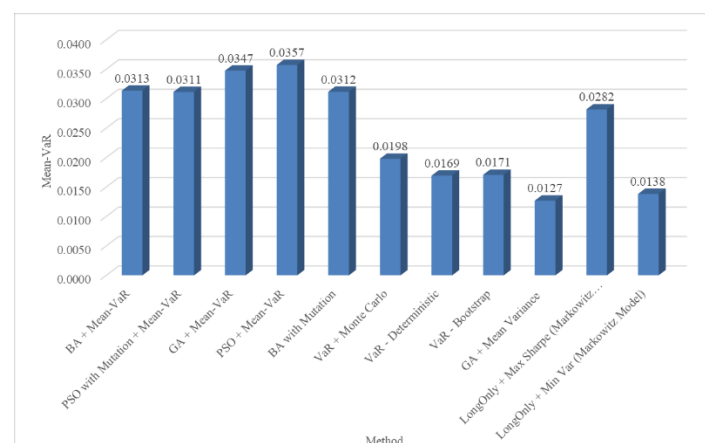


Figure 6. The average of mean-VaR by resulted by portfolio optimization methods

Figure 5 shows the average daily expected return of each method. The highest return was achieved by PSO + Mean-VaR (0.00230), followed by GA + Mean-VaR (0.00216) and the proposed PSO with Mutation + Mean-VaR (0.00202). BA with Mutation (0.00171), BA + Mean-VaR (0.00159), and the Markowitz Max Sharpe model (0.00152) achieved moderate returns. In contrast, VaR with Monte Carlo

(0.00055), GA + Mean Variance (0.00072), and the Markowitz Min Var model (0.00031) produced the lowest returns.

Building on this, Figure 6 reports the average Mean-VaR values, which capture downside risk. The lowest value was obtained by PSO with Mutation + Mean-VaR (0.0311), followed by BA with Mutation (0.0312) and BA + Mean-VaR (0.0313). Higher values were recorded by GA + Mean-VaR (0.0347) and PSO + Mean-VaR (0.0357). Other methods, such as VaR–Monte Carlo (0.0198), VaR–Deterministic (0.0169), VaR–Bootstrap (0.0171), GA + Mean Variance (0.0127), and the Markowitz models (0.0282 and 0.0138), produced comparatively lower levels.

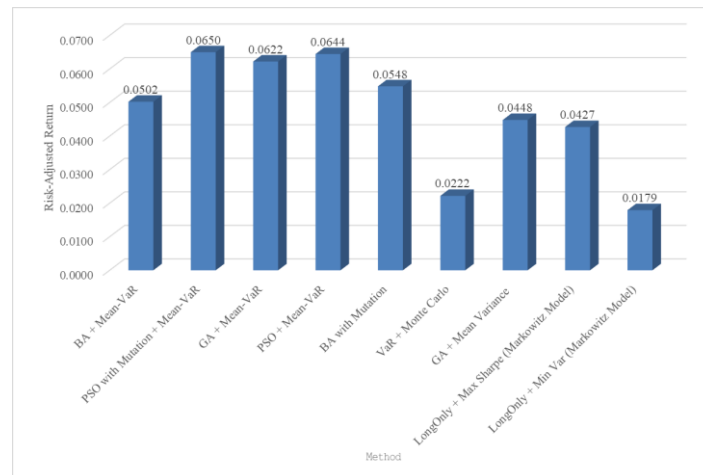


Figure 7. The average of risk-adjusted return by resulted by portfolio optimization methods

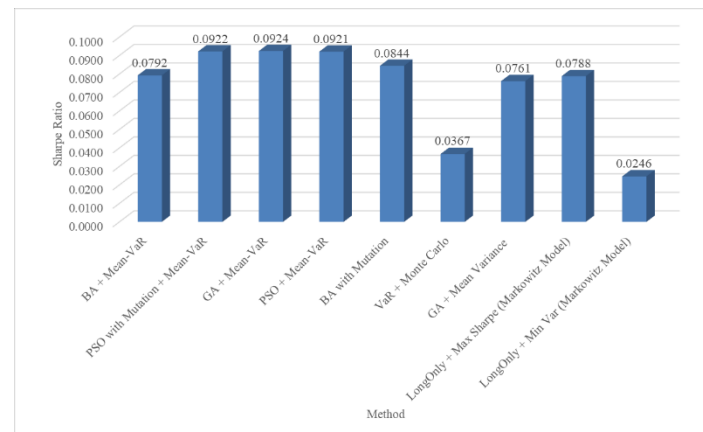


Figure 8. The average of sharp ratio by resulted by portfolio optimization methods

Figure 7 then presents the risk-adjusted return values. The highest result was achieved by PSO with Mutation + Mean-VaR (0.0650), followed by PSO + Mean-VaR (0.0644) and GA + Mean-VaR (0.0622). BA with Mutation (0.0548) and BA + Mean-VaR (0.0502) recorded moderate values, while lower results were obtained by GA + Mean Variance (0.0448), LongOnly + Max Sharpe (0.0427), VaR + Monte Carlo (0.0222), and the Markowitz Min Var model (0.0179).

Continuing with performance ratios, Figure 8 compares the Sharpe Ratios across methods. The highest ratio was achieved by GA + Mean-VaR (0.0924), followed closely by PSO with Mutation + Mean-VaR (0.0922) and PSO + Mean-VaR (0.0921). BA with Mutation (0.0844) and BA + Mean-VaR (0.0792) recorded slightly lower values. Smaller ratios were observed for GA + Mean Variance (0.0761), LongOnly + Max Sharpe (0.0788), VaR + Monte Carlo (0.0367), and the Markowitz Min Var model (0.0246).

Finally, Figure 9 shows the average execution time of each method. The slowest was GA + Mean-VaR (8.004 s), followed by PSO with Mutation + Mean-VaR (7.514 s) and GA + Mean Variance (7.382 s). PSO + Mean-VaR (4.628 s) and BA + Mean-VaR (4.247 s) recorded shorter times, while BA with Mutation required 6.067 s. In contrast, the fastest methods were the Markowitz Min Var (0.218 s), Markowitz Max Sharpe (0.224 s), and VaR + Monte Carlo (0.327 s).

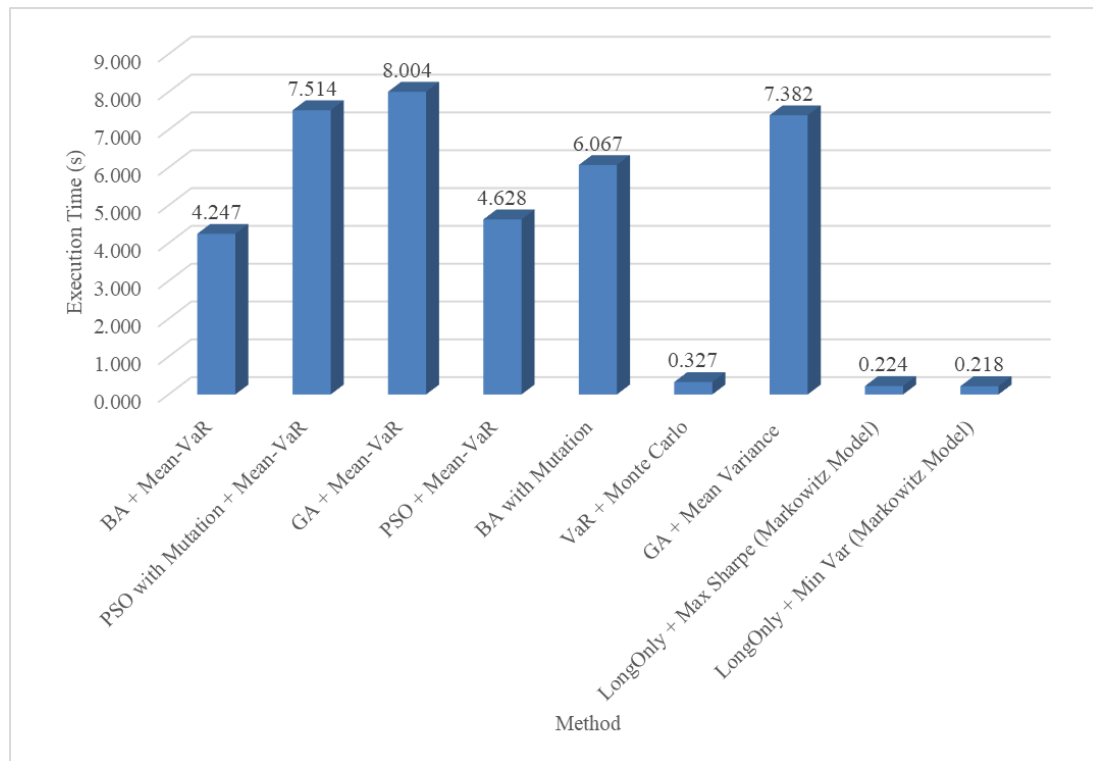


Figure 9. The average of execution time by resulted by portfolio optimization methods

Table 4. Standard deviation performance metrics over 25 runs.

No.	Methods	Expected Return	Mean-VaR	Risk-Adjusted Return	Sharpe Ratio	Execution Time (s)
1	BA + Mean-VaR	0.000	0.004	0.007	0.008	1.711
2	PSO with Mutation + Mean-VaR	0.000	0.002	0.000	0.000	2.591
3	GA + Mean-VaR	0.002	0.035	0.062	0.092	8.004
4	PSO + Mean-VaR	0.002	0.036	0.064	0.092	4.613
5	BA with Mutation	0.002	0.031	0.055	0.084	5.817
6	VaR + Monte Carlo	0.001	0.000	0.001	0.002	0.203
7	GA + Mean Variance	0.000	0.016	0.002	0.003	3.274
8	LongOnly + Max Sharpe (Markowitz Model)	0.000	0.003	0.001	0.002	0.074
9	LongOnly + Min Var (Markowitz Model)	0.000	0.001	0.005	0.010	0.067

Since average values alone may not fully capture algorithm reliability, Table 4 reports the standard deviations of the performance metrics over 25 independent runs. The results show the variability in expected return, Mean-VaR, risk-adjusted return, Sharpe ratio, and execution time for each method. PSO with Mutation + Mean-VaR exhibited consistently low deviations across most metrics, indicating stable and repeatable performance. In contrast, some other metaheuristic methods showed larger fluctuations, reflecting less consistent outcomes. Overall, the table highlights the robustness of the proposed algorithm in maintaining reliable results across multiple executions.

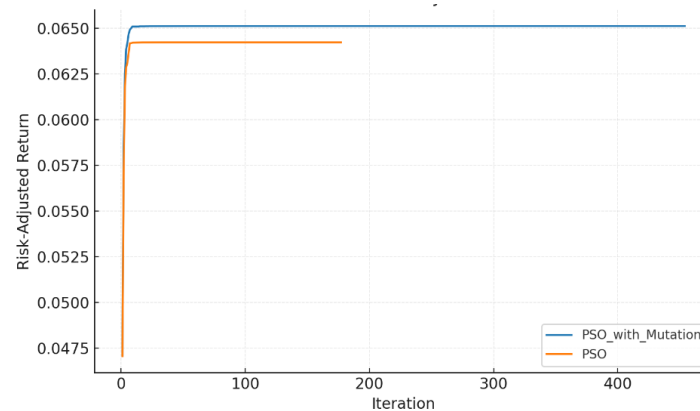


Figure 10. Convergence curves (best fitness per iteration): PSO vs PSO with Mutation

To complement the summary of average performance metrics, a convergence plot is presented to illustrate how the algorithms evolve during optimization in Figure 10. The plot shows the best-so-far risk-adjusted return as a function of iteration for PSO and PSO with Mutation. Both curves rise sharply in the first 10–20 iterations, indicating rapid early improvement. After that, PSO with Mutation overtakes and converges to a higher plateau (~0.065), whereas standard PSO levels off earlier at a lower value (~0.064). This pattern suggests that the mutation operator maintains population diversity and prevents premature convergence, enabling continued exploration and incremental gains. The advantage of PSO with Mutation is sustained across the full run (454 vs. 177 iterations recorded), demonstrating better final solution quality and stability in terms of risk-adjusted return.

4. DISCUSSIONS

The preprocessing stage played an important role in shaping the reliability of this study. The exclusion of five tickers with substantial missing data (Figure 3) ensured comparability across assets and prevented interpolation bias that could distort return and risk calculations. By retaining only 31 complete series, the optimization process was based on consistent inputs, improving the validity of the performance evaluation across methods.

The distribution of daily returns provided further justification for the methodological choices. The histogram of BBNI.JK returns (Figure 4) revealed negative skewness and leptokurtosis, with extreme losses occurring more frequently than expected under a normal distribution. These non-normal features highlight why conventional mean–variance models are insufficient and confirm the relevance of adopting downside-sensitive risk measures such as VaR and Mean-VaR. They also underscore the need for optimization approaches capable of handling heavy tails, such as the hybrid metaheuristics evaluated in this study.

Building on these foundations, the portfolio optimization results offer several important insights. Although PSO + Mean-VaR and GA + Mean-VaR produced slightly higher average returns (0.00230 and 0.00216 in Figure 5), they also carried higher downside risk, with Mean-VaR values of 0.0357 and 0.0347 in Figure 6, and showed greater variability across runs (Table 4). This highlights a weakness of

standard metaheuristics: they often focus on maximizing returns but expose portfolios to bigger risks, which is dangerous in volatile markets. In comparison, the proposed PSO with Mutation + Mean-VaR achieved a competitive return (0.00202), the lowest Mean-VaR (0.0311 in Figure 6), and more consistent results across multiple runs (Table 4). These outcomes indicate its stronger ability to limit losses during extreme market conditions, making it more reliable for real-world financial risk management.

A closer look at the algorithmic behaviour highlights the role of the mutation operator in driving these improvements. Standard PSO is known for fast convergence but often suffers from premature convergence, limiting exploration. By introducing controlled randomness, mutation preserves population diversity, enhances exploration, and reduces the risk of stagnation in local optima. This mechanism explains why PSO with Mutation consistently reached stronger trade-offs between return and risk. Although it sacrificed a small fraction of raw return compared to GA and standard PSO (Figure 5), the improved risk profile and stability (Figure 6 and Table 4) more than compensated for this trade-off, aligning well with the principle of risk-sensitive portfolio optimization.

The evaluation of risk-adjusted metrics further reinforces this interpretation. PSO with Mutation + Mean-VaR recorded the highest risk-adjusted return (0.0650 in Figure 7) and one of the strongest Sharpe Ratios (0.0922 in Figure 8), nearly matching GA (0.0924). Combined with its lower variability (Table 4), this shows the method's ability to generate reliable gains per unit of risk exposure. Although execution time increased moderately (7.514 seconds versus 4.628 for PSO and 4.247 for BA in Figure 9), this trade-off is acceptable given the significant improvements in robustness, stability, and downside risk control.

Additional evidence comes from the convergence analysis (Figure 10). Both PSO and PSO with Mutation improved rapidly in the first iterations, but the mutation operator allowed the latter to continue exploring the solution space, preventing premature convergence and enabling it to reach a higher plateau in terms of risk-adjusted return. This behaviour highlights the practical advantage of mutation in maintaining diversity and sustaining improvements over the entire optimization process.

These findings are consistent with prior studies [35], [36], [37], [38], which emphasize the advantages of hybrid metaheuristics for portfolio optimization under downside risk measures. However, the present study goes further by demonstrating that the proposed PSO with Mutation + Mean-VaR not only improves the portfolio return–risk profile but also delivers the lowest standard deviations across multiple runs (Table 4), showing its ability to combine strong performance with stability.

The key novelty of this study lies in integrating a mutation operator into PSO for Mean-VaR portfolio optimization in the context of an emerging market dataset (Indonesian banking sector). Unlike previous works that mainly evaluated GA, BA, or standard PSO, this study shows that mutation-enhanced PSO achieves a superior balance between return, downside risk, and stability, while also maintaining acceptable execution times. This contribution extends the literature by highlighting the robustness of hybrid PSO approaches in real-world, high-volatility environments.

From a practical standpoint, PSO with Mutation emerges as a strong candidate for portfolio optimization in volatile and uncertain environments such as the Indonesian banking sector. Its ability to deliver competitive returns (Figure 5), minimize downside risk (Figure 6), and maintain acceptable computational efficiency (Figure 9) makes it highly relevant for institutional investors and automated trading systems. The application to 31 Indonesian bank stocks over six years further demonstrates its scalability and real-world applicability in emerging markets, where volatility is often more pronounced.

From a scientific perspective, this study contributes to the broader field of computational intelligence by providing evidence of the effectiveness of hybrid metaheuristics in solving non-convex and complex optimization problems. Beyond finance, the success of integrating mutation into PSO validates the potential of hybrid approaches in domains such as scheduling, resource allocation, and

machine learning model training, where avoiding premature convergence and ensuring search diversity are equally critical.

Nevertheless, several limitations must be acknowledged. The analysis was conducted exclusively on the Indonesian banking sector, which may limit generalizability to other sectors or markets with different dynamics. Moreover, the evaluation was based on a fixed historical period, without stress testing under alternative economic scenarios. Future research could address these gaps by applying PSO with Mutation to multi-sector or multi-country datasets, incorporating stress-testing frameworks, and comparing its performance against other advanced hybrid metaheuristics or deep learning-based optimization approaches.

5. CONCLUSION

This study evaluated portfolio optimization using the proposed Enhanced PSO with Mutation (PSO with Mutation + Mean-VaR), benchmarked against both classical models and several Mean-VaR optimization approaches using metaheuristics. Performance was assessed across expected return, Mean-VaR, risk-adjusted return, Sharpe ratio, execution time, and variability over 25 runs. The results demonstrate that although PSO + Mean-VaR and GA + Mean-VaR achieved slightly higher average returns, they incurred greater downside risk. In contrast, the proposed method delivered a competitive return (0.0020), the lowest Mean-VaR (0.0311), the highest risk-adjusted return (0.0650), and a strong Sharpe ratio (0.0922), while also recording the lowest variability across all metrics. Although execution time was moderately higher (7.514 s), it remained more efficient than GA (8.004 s), confirming a good balance between efficiency and solution quality. Overall, these results confirm that PSO with Mutation offers the best overall trade-off, combining strong returns, effective downside risk management, computational efficiency, and consistency. The algorithm thus emerges as a reliable and practical solution for portfolio optimization, particularly in volatile financial markets where balancing profitability and risk control is essential. This study underscores the potential of hybrid metaheuristics to address real-world financial optimization challenges, especially in volatile and uncertain markets. Beyond the financial domain, it also highlights the relevance of mutation-enhanced PSO for computer scheduling, resource allocation, and machine learning model training, as a powerful, adaptive, and data-driven tool for solving complex non-convex optimization problems and supporting informed decision-making in dynamic environments. Future research can extend this study by applying the proposed PSO with Mutation to multi-objective portfolio optimization frameworks incorporating other risk measures, such as Conditional Value-at-Risk (CVaR) and drawdown constraints. Additionally, hybridizing PSO with deep learning models for predictive return estimation could further enhance decision-making in rapidly changing markets. Real-time adaptive parameter tuning and testing on high-frequency trading datasets may also improve algorithm responsiveness. Finally, expanding the evaluation to include different asset classes and cross-market datasets would help validate the algorithm's generalizability and robustness in diverse financial contexts.

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